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# Pulse Dynamics in a Model of Coupled Excitable Fibers

— A Variety of Patterns and Spatio-temporal Chaos —

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## §1. Introduction

It has been identified that there is a rich variety of interactions among spatially localized patterns such as pulses and spots in a reaction-diffusion medium.<sup>1)</sup> In addition, interactions among patterns in different reaction-diffusion media<sup>2)</sup> should be also of interests from practical viewpoints. In fact, in several nerve systems, it is observed that huge nerve axons are arranged in a densely packed bundle so that pulses traveling in adjacent axons electronically communicate each other.<sup>3)</sup> In this paper, we investigate what kinds of pulse dynamics can be emerged when two excitable reaction-diffusion media are coupled with each other, especially focusing on a situation in which parameters of two excitable fibers are not equal.<sup>4)</sup> Such a situation is not uncommon because the diameters of fibers are not equivalent in the real nerve systems in general, leading to a difference of diffusion coefficients in mathematical models.

## §2. Model

As an illustrative example of coupled excitable fibers, we consider the following mutually coupled one-dimensional FitzHugh-Nagumo (FHN) equations:

$$\begin{cases} \dot{u}_1 = u_1(u_1 - \alpha)(1 - u_1) - v_1 + \kappa_1 \nabla^2 u_1 + \epsilon(u_2 - u_1) \\ \dot{v}_1 = \tau(u_1 - \gamma v_1), \\ \dot{u}_2 = u_2(u_2 - \alpha)(1 - u_2) - v_2 + \kappa_2 \nabla^2 u_2 + \epsilon(u_1 - u_2) \\ \dot{v}_2 = \tau(u_2 - \gamma v_2). \end{cases} \quad (2.1)$$

Subscripts “1” and “2” denote the first and the second fibers. The state variables  $u_{1,2} = u_{1,2}(x, t)$  and  $v_{1,2} = v_{1,2}(x, t)$ , where  $x \in [0, L]$  and  $t \in [0, \infty)$  are space and time coordinates, are the activator and the inhibitor, respectively. The parameters  $\kappa_1$  and  $\kappa_2$  are diffusion coefficients. The value of  $\kappa_1$  is fixed as 0.25 throughout this paper. The mutual interaction between two excitable fibers is also introduced as the linear coupling terms  $(u_{1,2}(x, t) - u_{2,1}(x, t))$  with the strength  $\epsilon$  for activators. We take  $\epsilon$  and  $\kappa_2$  as the control parameters. The periodic boundary condition is employed.

## §3. Simulation Results

### 3.1. A Variety of Patterns

We investigate pulse dynamics when a right-moving pulse is initiated in fiber 1 and fiber 2 is set to the global resting state, as initial conditions. The parameters of the reaction kinetics is fixed as  $\alpha = 10^{-1}$ ,  $\gamma = 2.5$ , and  $\tau = 2 \times 10^{-3}$  so that local dynamics exhibits an excitable property.

#### Soliton-like Pulse Collision

Focus on the case in which diffusion coefficients are different:  $\kappa_1 = 0.25$  and  $\kappa_2 = 0.09$ . When the inter-fiber coupling strength  $\epsilon$  crosses over a threshold value, a propagating pulse in

fiber 1 can induce an excitation in fiber 2. A pulse in fiber 2, however, cannot induce an excitation in fiber 1. This “one-way” excitation is understood as follows. In general, the existence of the diffusion term  $\nabla^2 u$  in the equation of the activator suppresses the excitation by an external stimulus. Because we consider the case  $\kappa_1 > \kappa_2$  in Eqs. (2.1) now, a more intensive stimulus from fiber 2 is required for inducing an excitation in fiber 1. A typical pattern for  $\epsilon = 8 \times 10^{-3}$  caused by this one-way excitation is shown in Fig. 1:

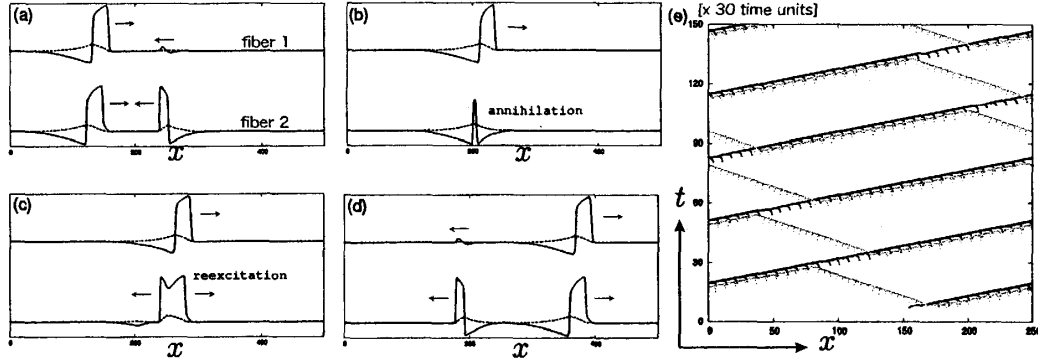


Fig. 1. (a-d) A series of snapshots for the soliton-like pulse collision. (e) A spatio-temporal plot.

- (a) Head-on collisions occur in both fibers.
- (b) Supra-threshold pulses annihilate each other in fiber 2 through the collision, whereas the sub-threshold pulse in fiber 1 does not significantly affect the propagation of supra-threshold pulse in fiber 1.
- (c) A new supra-threshold excitation is induced in fiber 2 by the pulse in fiber 1, and it splits into two pulses propagating in opposite directions.
- (d) All profiles are recovered after head-on collisions like *solitons*.

#### Recombination of Synchronized Pulses

In Fig. 1 (a), two pulses are facing with each other in fiber 2. By changing initial conditions, we can prepare two pulses propagating in the same direction in the fiber 2 as shown in Fig. 2 (a). We find the following dynamical behavior associated with the destruction of synchronized pulses as shown in Fig. 2:

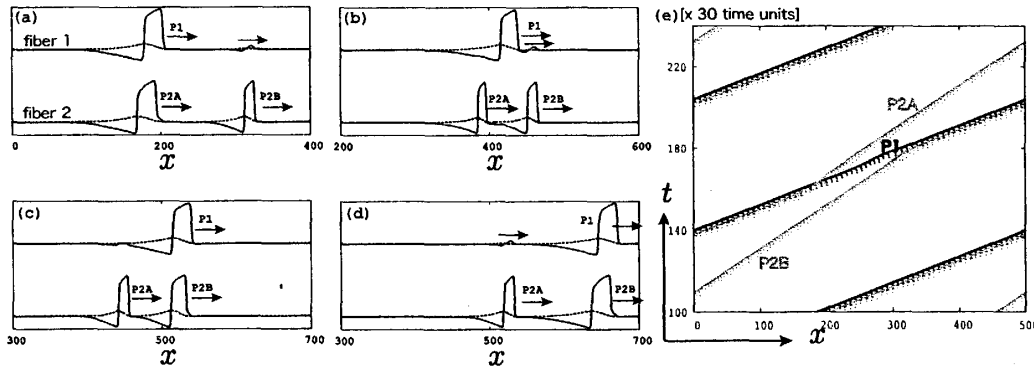


Fig. 2. (a-d) A series of snapshots for the recombination process. (e) A spatio-temporal plot.

- (a) A pair of synchronized pulses P1 and P2A becomes close to P2B.
- (b) Synchronization is broken by the highly concentrated region of the inhibitor behind P2B.
- (c) A new pair of synchronized pulses composed of P1 and P2B forms.
- (d) The new synchronized pulses move away from P2A.

We call dynamical processes *recombination* of synchronized pulses.

### 3.2. Spatio-temporal Chaos

The following parameters  $\alpha = 5 \times 10^{-3}$ ,  $\gamma = 0.5$ , and  $\tau = 5 \times 10^{-3}$  admit the “oscillating wake” of propagating pulse, since the eigenvalues of the Jacobi matrix at the resting point  $(u, v) = (0, 0)$  has an imaginary part. For such parameter values of the reaction kinetics, interesting dynamical behaviors are observed.

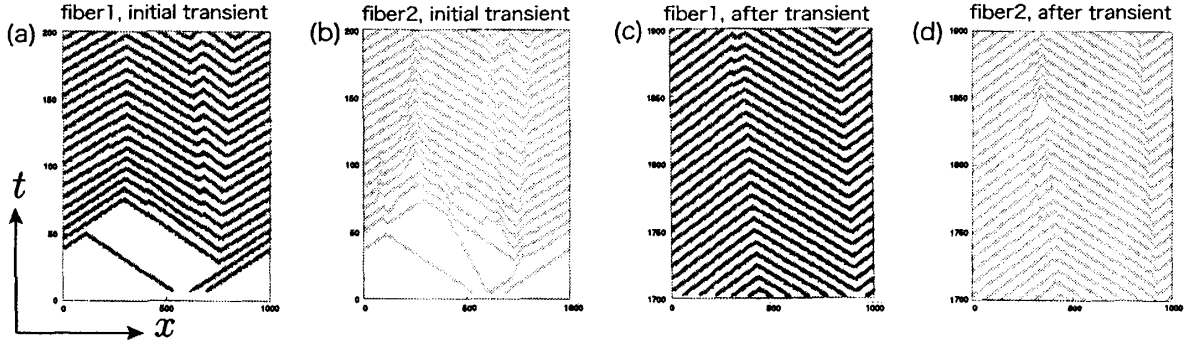


Fig. 3. Spatio-temporal plots at initial stage in (a-b), and after transient in (c-d).  $L = 10^3$ .

Figures 3 (a) and (b) show spatio-temporal patterns at initial stage of the system of Eqs (2-1) for  $\Delta\kappa = 0.218$  and  $\epsilon = 5 \times 10^{-3}$ . If  $\Delta\kappa = 0$ , a stable reentrant wave is observed for  $\epsilon = 5 \times 10^{-3}$ . In Fig. 3 (a), the “V”-shaped structure corresponds to the event of a supra-threshold excitation splitting into two propagating pulses and the “^”-shaped structure corresponds to the event of an annihilation between two pulses. It is seen from Fig. 3 (a) that two sources of splitting pulses are generated at initial stage. After transient, however, only one of the two remains as shown in Figs. 3 (c) and (d).

In Fig. 3 (d), some disordered defects are observed near the annihilation locations even after a transient died out. In order to understand the origin of such disordered structures, we investigate the dependence of the return map  $\eta(t)$  versus  $\eta(t + t_s)$  on  $\Delta\kappa$  after a transient, where  $\eta(t)$  is a spatially coarse-grained variable:  $\eta(t) = \sqrt{(1/L) \int_0^L |u_1(x, t) - u_2(x, t)|^2 dx}$ . Results are shown in Figs. 4 (a-c), and corresponding power spectra are also plotted in their insets. Here, we take  $t_s = 30$ . When  $\Delta\kappa$  is small, the return map shows a closed curve, which indicates that the dynamics of  $\eta(t)$  is a periodic motion. For larger  $\Delta\kappa$ , the dynamics of  $\eta(t)$  becomes more complicated, and the change of results from Fig. 4 (a) to Fig. 4 (c) suggests that the system shows a quasiperiodicity route to chaos.

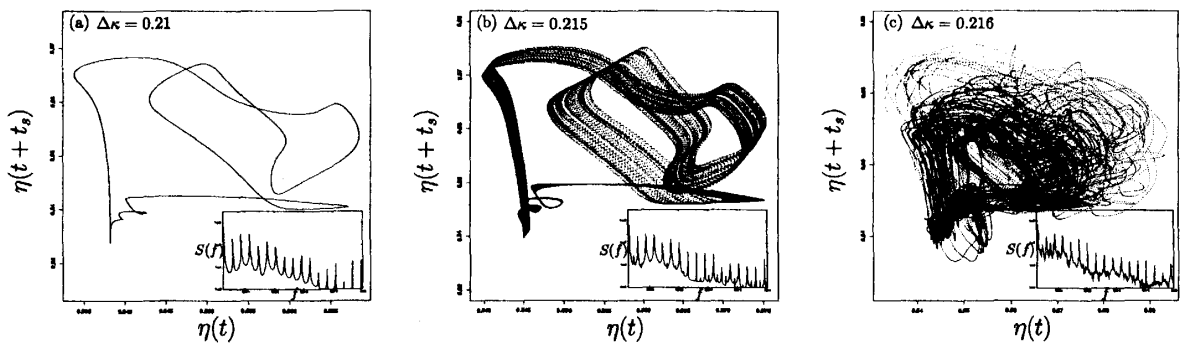


Fig. 4. (a-c) The graphs of  $\eta(t)$  vs.  $\eta(t + t_s)$  after a transient for three different values of  $\Delta\kappa$ . Corresponding power spectra are also shown in insets.

### References

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